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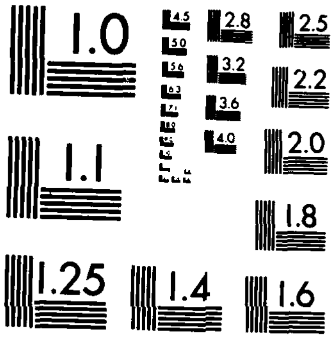

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THERMAL RUNAWAY DUE TO STRAIN-HEATING FEEDBACK

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Abstract

A one-dimensional, dynamic, thermomechanical model, which includes nonlinear inelastic deformation, internal heat generation (strain-heating), temperature dependent material properties, thermal expansion and thermoelastic coupling, is considered for a uniform thin bar subjected to mechanical or thermal disturbance. A nonlinear Maxwell material is examined in this model and special attention is focused on the temperature change. By solving a nonlinear problem, it is found that a thermal instability, called thermal runaway, may result due to the mutual feedback between strain-heating and the temperature dependent inelastic material properties. Neglecting this important phenomenon may lead to unexpected material failure. A linearizing perturbation study then shows that the occurence of this instability depends on the choice of the material, the steady state values of stress and temperature and on the characteristic length of the bar, rather than on the magnitude or the form of the disturbance. It is also found that thermal expansion, inertia and thermoelastic coupling have relatively minor effect on this instability for the problem under consideration. Aluminum is taken as an example for numerical demonstration.

1. Introduction

It is well-known from thermodynamics that inelastic deformations may result in energy dissipation (Boley and Weiner 1960, Malvern 1969). The conversion of dissipative mechanical energy into heat is known as strain-heating, and this phenomenon has been studied theoretically by Allen (1985) for viscoplastic materials and by Tauchert (1967a) for viscoelastic materials. It has also been studied experimentally for various materials (Tauchert 1967b, Dillon 1962a,b,1966) and significant temperature increases have been observed, e.g. a 500°K temperature rise was measured (Dillon 1962a). However, few comprehensive studies have been carried out to explore the conditions under which a significant temperature rise may result. It is therefore our objective here to include a wide spectrum of thermal effects, such as thermal expansion, thermoelastic coupling, temperature dependence of (inelastic) material properties and strain-heating, in a one-dimensional, dynamic, thermomechanical model. Then based on this rather general model, a mathematical study of the effect of mechanical or thermal disturbance is given for a nonlinear Maxwell material with temperature dependent viscosity. This type of material is chosen because of the simple form of the constitutive law and because of its widespread use in the literature to approximate the behavior of some polymers and metallic materials at elevated temperatures.

An important phenomenon known as thermal runaway, which is an unstable feedback process, is observed. This thermal instability has been studied analytically by Schapery (1964, 1965), Huang and Lee (1967) and experimentally by Schapery and Cantey (1966) for linear viscoelastic rods subjected to cyclic loading. It may occur as the temperature rise due to strain-heating results in a reduction of the inelastic material properties (e.g., the viscosity coefficient in the case of Maxwell material), i.e. the material softens, which

in turn generates greater inelastic strain and strain-heating. It is also found that thermal expansion, thermoelastic coupling and inertia have a relatively minor effect on the occurence of this instability. The thermal runaway phenomenon has been discussed in the geophysics literature (e.g. Brun and Cobbold 1980, Cary et al. 1979 and Wan et al. 1986), but has not received much attention in the engineering community. This is probably due to the fact that thermal dissipative mechanisms, such as convection, are often present in conventional engineering applications such that thermal runaway is not a serious problem. However, in the absence of such thermal dissipative mechanisms (e.g., in space) thermal runaway could lead to unexpected failure.

In this paper, we will study a thermo-mechanical system of governing equations for a uniform thin bar with one end fixed and insulated, and with the other end subject to a mechanical or thermal disturbance. Thermal expansion and thermoelastic coupling, which are well-known effects in coupled thermoelasticity (Boley and Weiner 1960) will be included in the material constitutive law and in the energy equation, respectively. Furthermore, the energy equation will contain the strain-heating term. These two equations, together with the conservation of momentum and strain-displacement relation, form a set of coupled nonlinear partial differential equations which govern the response of the system. Particular emphasis will be placed on the evaluation of the temperature changes.

This nonlinear problem is first solved in Section 2 with the assumption that the load is applied so slowly that the inertial effect can be ignored. The resulting problem is decoupled mathematically, and thus the temperature field can be determined by an iterative procedure with the use of a Green's function. The solution to this nonlinear problem clearly demonstrates the thermal runaway phenomenon. Next, employing the standard perturbation

technique, linearized equations, which are amenable to a stability analysis, are obtained and studied in Section 3. Solutions are obtained by the method of separation of variables, and then the stability analysis is performed. Analytical solutions under quasi-static conditions are obtained both for the case of a mechanical disturbance and for a thermal disturbance. Numerical results are presented for an aluminum bar, and some concluding remarks are given in Section 4.

2. The Nonlinear Problem

Consider a thin bar length L with one end ($x = 0$) fixed, and with the other end ($x = L$) subject to an axial stress load p(t); the bar is initially at a uniform temperature T_o. The constitutive law for a very wide class of inelastic materials may be expressed as

$$Q(\varepsilon) = P(\sigma) + R(T) + g(\sigma,\varepsilon;\ T) \tag{1}$$

where $\varepsilon(x,t)$, $\sigma(x,t)$ and $T(x,t)$ are respectively the normal strain, normal stress and temperature. These terms in general vary with the axial coordinate x and the time t. In equation (1), P, Q, and R are linear differential operators in time and g is a general nonlinear fucntion, which are chosen to characterize the viscoelastic or viscoplastic behavior of the material. For a fairly large class of such materials, equation (1) can be simplified to

$$\frac{\partial\varepsilon}{\partial t} = \frac{1}{E}\frac{\partial\sigma}{\partial t} + \alpha\frac{\partial T}{\partial t} + f(\sigma,\varepsilon;T) \tag{2}$$

The first term and the second term on the right hand side of equation (2) represent, respectively, the instantaneous linear elastic response and the thermal expansion, which are characterized by Young's modulus, E, and by the linear thermal expansion coefficient, α. The last term in equation (2) is the inelastic strain rate which depends directly on σ and ε with the dependence on T occurring indirectly through the temperature dependence of the material properties. Various forms for f have been postulated for different materials and for different loading situations (e.g. see Cristescu 1967, Cernocky and Krempl 1980).

The conservation of energy equation consistent with equation (2) (e.g. see Chang and Cozzarelli 1977 for nonlinear thermoviscoelastic materials) is given

as

$$k \frac{\partial^2 T}{\partial x^2} + \sigma f(\sigma,\varepsilon;T) - \alpha T_o \frac{\partial \sigma}{\partial t} = \rho c \frac{\partial T}{\partial t} \tag{3}$$

where k, ρ and c are the thermal conductivity, density and heat capacity of the material, respectively. The second term in equation (3) is known as the strain-heating term, whereas the third term represents the thermoelastic coupling effect. Note that heat flux through the lateral surface has been neglected, i.e., the lateral surface of the bar is assumed to be insulated. Also, it is assumed that all of the mechanical energy during the deformation process is transformed into heat. Equations (2) and (3), together with the conservation of momentum

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{4}$$

and the strain-displacement relation

$$\varepsilon = \frac{\partial u}{\partial x} \tag{5}$$

where u(x,t) is the axial displacement, form a set of equations which completely describe the response of the system.

In particular, a nonlinear Maxwell model with temperature dependent viscosity will be considered here. For this type of material, equations (2-4) can be rewritten with the use of equation (5) as

$$\frac{\partial v}{\partial x} = \frac{1 \partial \sigma}{E \partial t} + A(T)\ \sigma^n + \alpha \frac{\partial T}{\partial t} \tag{6}$$

where

$$A(T) = A_o e^{-B/T}$$

is the reciprocal viscosity,

$$k \frac{\partial^2 \dot{}}{\partial x^2} + A(T)\ \sigma^{n+1} - \alpha T_o \frac{\partial \sigma}{\partial t} = \rho c \frac{\partial T}{\partial t} \tag{7}$$

and

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \tag{8}$$

The variable $v(x,t)$ is the axial velocity, and A_o, B and n are respectively the pre-exponential constant in the reciprocal viscosity, the creep activation constant and the stress power. Equations (6-8) represent a system of coupled, nonlinear partial differential equations for a nonlinear Maxwell material in terms of the dependent variables v, σ, and T. In general, it is difficult to solve these equations because of the presence of strong nonlinearities and coupling.

Now, let us consider a special case in which the characteristic time, t_o, of the applied stress is large enough so that inertia effects can be neglected. It follows immediately from equation (8) that the stress field is uniformly distributed and simply equal to the applied stress at the free end. As a result, the problem is uncoupled and thus the temperature and velocity fields can be obtained successively. As an example, it is assumed that the bar is insulated at the fixed end and maintained at a constant temperature, T_o, at the free end. The Green's function associated with the auxiliary problem, in which the known heat source term in equation (7), (i.e., the second and the third terms) are suppressed; is simply

$$G(x,t|x',\tau) = \frac{2}{L}\sum_{m=1}^{\infty} \exp[-\frac{k}{\rho c}\beta_m^2(t-\tau)]\ \cos\beta_m x\ \cos\beta_m x' \tag{9}$$

where $\beta_m = \dfrac{(2m-1)\pi}{2L}$. If an initial guess for the solution is available, say

$T^{(0)}(x,t)$, then the following integrals (e.g. see Ozisik 1980)

$$T^{(i+1)}(x,t) = \frac{1}{\rho c} \int_{\tau=0}^{\tau=t} \int_{x'=0}^{x'=L} G(x,t|x',\tau) \, [A(T^{(i)}(x',\tau))p(\tau)^{n+1}$$

$$-\alpha T_o \frac{\partial p(\tau)}{\partial \tau}] \, dx'd\tau \qquad i = 0,1,2,\ldots\ldots \qquad (10)$$

can be carried out successively by numerical methods and will converge to the desired solution. The rate of convergence of this iterative process depends greatly on the initial guess. Usually, the solution for the completely insulated case (which can be easily calculated) will be a reasonable choice for the initial guess.

The solution for an aluminum bar subjected to an applied stress of the form

$$p(t) = \sigma_o \, (1-e^{-t/t_o}) \qquad (11)$$

at the free end is shown in Figs. 1 and 2. The material properties and other necessary parameters are given in Table 1; the creep data were taken from Garafalo (1967), Walter and Ponter (1976). Fig. 1 shows the temperature increase with time at $x = 0$, $L/2$ and $3L/4$, while Fig. 2 shows the temperature distribution over x at $t = 250$, 500, 750 and 1000 seconds. These two figures clearly indicate that without appropriate heat dissipating mechanisms, the temperature can increase without bound leading eventually to failure.

It is important to note that although the above problem has been decoupled mathematically by considering the quasi-static case, it is still coupled physically because the strain-heating feedback into the temperature dependent viscosity has already been imbedded in the energy equation and the material constitutive law. Also, it is found in this example that thermal expansion and thermoelastic coupling have a relatively minor effect on the temperature rise.

3. The Linearized Problem - Stability Analysis

It has been demonstrated in the previous section that the temperature can increase significantly due to strain-heating and may lead to eventual failure. To prevent this from occurring, a cooling process can be designed to extract the heat generated internally and thus control the temperature. Mathematically, this can be accomplished by introducing a heat sink term $S_o = -A(T_o)p(t)^{n+1}$, into the energy equation. As a result, the system will reach a steady state in which $T = T_o$, $\sigma = \sigma_o$ and $v = v_o x/L$ where $v_o = A(T_o)\sigma_o^{\,n}L$. In this section we assume that the system is initially in this steady state condition.

If the system is then subjected to a small additional external disturbance in stress or temperature at $x = L$, this disturbance will result in additional increments of stress, $\tilde{\sigma}$, temperature $\tilde{T}$, and velocity, $\tilde{v}$. It is assumed that the magnitudes in the steady state are much larger than the magnitudes of the increments in the perturbed state, and we may then write

$$\begin{aligned} T &= T_o + \tilde{T},\ T_o \gg \tilde{T} \\ \sigma &= \sigma_o + \tilde{\sigma},\ \sigma_o \gg \tilde{\sigma} \\ v &= v_o + \tilde{v},\ v_o \gg \tilde{v} \end{aligned} \tag{12}$$

Substituting equation (12) into equations (5-8), subtracting the relation for the steady state (with sink S_o added to equation (7)), retaining the linear terms in the increments (as in Cozzarelli et al. 1970), and then introducing the nondimensional quantities

$$\bar{T} = \frac{\tilde{T}}{T_o},\ \bar{\sigma} = \frac{\tilde{\sigma}}{\sigma_o},\ \bar{v} = \frac{\tilde{v}}{v_o},\ \bar{x} = \frac{x}{L},\ \bar{t} = \frac{t}{L/v_o},\ \bar{E} = \frac{E}{\sigma_o},\ \bar{\rho} = \frac{\rho}{\sigma_o/v_o^{\,2}}$$

$$\bar{\alpha} = \frac{\alpha}{1/T_o},\ \bar{k} = \frac{k}{Lv_o\sigma_o/T_o},\ \bar{c} = \frac{c}{v_o^{\,2}/T_o},\ \eta = \frac{B}{T_o},\ \bar{\sigma}_1 = \frac{\sigma_1}{\sigma_o},\ \bar{T}_1 = \frac{T_1}{T_o}$$

we obtain the following linearized equations:

$$\frac{\partial v}{\partial x} = \frac{1}{E}\frac{\partial \sigma}{\partial t} + n\sigma + \eta T + \alpha \frac{\partial T}{\partial t} \tag{13}$$

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \tag{14}$$

$$k \frac{\partial^2 T}{\partial x^2} + (n+1)\ \sigma + \eta T - \alpha \frac{\partial \sigma}{\partial t} = \rho c \frac{\partial T}{\partial t} \tag{15}$$

Note that the overbars ($\bar{\ }$) have been deleted in equations (13-15) for the sake of convenience of notation. The associated boundary and initial conditions are as follows:

$$\begin{aligned} &v = 0,\ \frac{\partial T}{\partial x} = 0 && \text{at } x = 0 \\ &\sigma = \sigma_1,\quad T = T_1 && \text{at } x = 1 \\ &T = \sigma = v = 0 && \text{at } t = 0 \end{aligned} \tag{16}$$

where $\sigma_1 = 0$ for a thermal disturbance and $T_1 = 0$ for a mechanical disturbance.

It can be seen that the method of separation of variables may be employed if we separate equations (13-16) into two problems (see Street 1973), i.e. a steady-state problem with inhomogeneous boundary conditions and a transient problem with homogeneous boundary conditions. Solving these two problems and superposing solutions, we obtain

$$T = T_i + \sum_{m=1}^{\infty} p_m(t)\ \cos\beta_m x \tag{17}$$

$$\sigma = \sigma_i + \sum_{m=1}^{\infty} q_m(t) \cos\beta_m x \tag{18}$$

$$v = v_i + \sum_{m=1}^{\infty} r_m(t) \sin\beta_m x \tag{19}$$

where

$$\beta_m = \frac{(2m-1)\pi}{2}$$

The temporal functions, $p_m(t)$, $q_m(t)$ and $r_m(t)$ are to be determined. In equations (17-19), T_i, σ_i and v_i are the following solutions for the steady state problem:

$$T_i = \frac{n+1}{\eta} \left(\frac{\cos\xi x}{\cos\xi} - 1\right) \sigma_1$$

$$\sigma_i = \sigma_1$$

$$v_i = \sigma_1 \left[-x+(n+1) \frac{\cos\xi x}{\xi\cos\xi}\right]$$

for a mechanical disturbance, and

$$T_i = T_1 \frac{\cos\xi x}{\cos\xi}$$

$$\sigma_i = 0$$

$$v_i = T_1\eta \frac{\sin\xi x}{\xi\cos\xi}$$

for a thermal disturbance, where

$$\xi = \left(\frac{\eta}{k}\right)^{1/2}$$

Substituting equations (17-19) into equations (13-15), multiplying by $\cos\beta_\ell x$, $\sin\beta_\ell x$ and $\cos\beta_\ell x$ (= 1,2,....), respectively, integrating the resulting equations over the space domain (x = 0 to x = 1) and invoking the orthogonality properties, the quantities $p_m(t)$, $q_m(t)$, and $r_m(t)$ in equations (17-19) can be solved approximately for m = 1,2,..,M. Accordingly, we obtain the formulation

$$\dot{\underline{Z}} = \underline{A}\,\underline{Z} \tag{20}$$

where

$$\underline{Z} = [p_1 p_2 \ldots . p_M | q_1 q_2 \ldots . q_M | r_1 r_2 \ldots . r_M]^T \qquad (3M\text{x}1)$$

and

$$\underline{A} = \begin{bmatrix} \rho c\underline{I} & \alpha\underline{I} & \underline{0} \\ \alpha\underline{I} & 1/E\underline{I} & \underline{0} \\ \underline{0} & \underline{0} & \rho\underline{I} \end{bmatrix}^{-1} \begin{bmatrix} \underline{C} & (n+1)\underline{I} & \underline{0} \\ -\eta\underline{I} & n\underline{I} & \underline{D} \\ \underline{0} & -\underline{D} & \underline{0} \end{bmatrix} \qquad (3M\text{x}3M)$$

where $\underline{I}$ is the identity matrix, and $\underline{C}$ and $\underline{D}$ are the diagonal matrices

$$\underline{C} = \mathrm{diag}(\eta - k\beta_m^2) \qquad m = 1,2,..,M$$

$$\underline{D} = \mathrm{diag}(\beta_m) \qquad m = 1,2,..,M$$

As initial conditions, we have

$$p_m(0) = (-1)^{m+1} \frac{(n+1)}{\eta} 2\beta_m \left(\frac{1}{\xi^2-\beta_m^2} + \frac{1}{\beta_m^2}\right) \sigma_1$$

$$q_m(0) = (-1)^{m+1} \frac{2}{\beta_m} \sigma_1$$

$$r_m(0) = (-1)^{m+1}\, 2\left(\frac{1}{\beta_m^2} + \frac{n+1}{\xi^2-\beta_m^2}\right) \sigma_1$$

for the mechanical disturbance, and

$$p_m(0) = (-1)^{m+1}\, 2\beta_m{}^2 \frac{1}{\xi^2-\beta_m{}^2} T_1$$

$$q_m(0) = 0$$

$$r_m(0) = (-1)^{m+1} \frac{2\eta}{\xi^2-\beta_m{}^2}$$

for the thermal disturbance. To obtain accurate results, a reasonable number of terms in equations (17-19) (e.g. $M \geq 6$) should be considered. However, this will introduce difficulties in the numerical integrations, because matrix $\underline{A}$ is of relative large order. Also, because of the inertia effect the matrix $\underline{A}$ is stiff, and thus a very small time step has to be taken for each integration. These two facts make it very difficult to obtain long time solutions by direct integration. Alternatively, transform methods (e.g., see Meirovitch 1980, Chen 1984) may be employed to solve equation (20).

If as in the nonlinear problem, we again neglect the inertia effect, the solution of equations (13-15) may be found with the use of variable transformations (Ozisik 1980, Carslaw and Jaegger 1959) as

$$T(x,t) = \frac{2(n+1)}{\rho c} \sigma_1 \sum_{m=1}^{\infty} \frac{(-1)^m}{\beta_m \lambda_m} (1-e^{\lambda_m t}) \cos\beta_m x \tag{21}$$

for the mechanical disturbance and

$$T(x,t) = \frac{\cos\xi x}{\cos\xi} T_1 + \sum_{m=1}^{\infty} (-1)^{m+1} 2\beta_m T_1 \frac{1}{\xi^2-\beta_m{}^2} e^{\lambda_m t} \cos\beta_m x \tag{22}$$

for thermal disturbance. In equations (21) and (22),

$$\beta_m = \frac{2m-1}{2}\pi$$

$$\lambda_m = \frac{1}{\rho c} (\eta - k\beta_m^2)$$

It is interesting to note that the same λ_m appears in equations (21) and (22). This indicates that the perturbations in temperature will follow the same general pattern, regardless of the forms and the magnitudes of the disturbances. Furthermore, for this quasi-static solution to be stable, it is required that

$$\xi^2 = \frac{\eta}{k} < \beta_1^2 \tag{23}$$

or in dimensional form

$$\frac{BL^2 A_o e^{-B/T_o} \sigma_o^{n+1}}{T_o^2 k} < \frac{\pi^2}{4} \tag{24}$$

Using the values of the parameters listed in Table 1, it is found that conditions (23) or (24) are violated. Thus, the quasi-static system with these values for the parameters is unstable.

Let us now return to a consideration of stability for the dynamic problem. Substituting equations (17-19) into equations (13-15), and combining the resulting equations into one equation in terms of p_m gives

$$a_1 \dddot{p}_m + a_2 \ddot{p}_m + a_3 \dot{p}_m + a_4 p_m = 0 \tag{25}$$

where

$$a_1 = \frac{1}{E} (\rho c - \alpha^2 E)$$

$$a_2 = \frac{k}{E} \beta_m^2 - \frac{\eta}{E} + \rho c n + \alpha(n+1) - \alpha\eta \tag{26}$$

$$a_3 = k\beta_m^2 n + c\beta_m^2 + \eta$$

$$a_4 = \frac{1}{\rho} (k\beta_m^2 - \eta) \beta_m$$

The overdots, ($\dot{}$), represent derivatives with respect to the time. By applying the Routh-Hurwitz criterion (e.g., see Kuo 1982), one root (i.e. $m = 1$) with a positive real part is found for aluminum using the parameters listed in Table 1. This observation indicates that inertia, thermal expansion and thermoelastic coupling, which were not considered in obtaining equations (21) and (22), do not contribute to the instability found earlier. Thus, it may be concluded that the mechanisms which may cause thermal instability (runaway) are the temperature dependence of inelastic material properties and the strain-heating. This conclusion can also be obtained by letting $B = 0$ (or $\eta = 0$ in nondimensional form) in equation (26). The resulting coefficients in equation (25) are then all positive (see Chang and Cozzarelli 1977 for $a_1 > 0$) and

$$a_2 a_3 - a_1 a_4 = \left(\frac{k}{E} n + \frac{\alpha^2}{\rho}\right) k\beta_m^{\ 4} + [\rho c n^2 k + \rho c^2 + \alpha(n+1)\ kn$$
$$+ \alpha(n+1)c]\beta_m^{\ 2} > 0$$

Using the Routh-Hurwitz criterion again, the system is seen to be stable in this case. Thus, if the temperature dependence of the material properties is not considered, the thermal runaway phenomenon will not be observed.

It is important to note that in accordance with condition (23) the occurrence of thermal instability depends on one nondimensional parameter, ξ, whose value is determined by the material properties A_o, B, n, k, the steady state values T_o, σ_o, and the length of the bar L. Thus, in the linearized problem the occurrence of runaway is not affected by the magnitude or form of the disturbance. The stability relationship between T_o and σ_o (i.e. condition (24)) for an aluminum bar of length 1 meter is shown in Fig. 3. Also shown in Figs. 4 and 5 are the temperature increments due to mechanical and thermal disturbances, respectively, where σ_1/σ_o and T_1/T_o are given at the end of Table 1.

4. Concluding Remarks

A one-dimensional, dynamic, thermomechanical model, which includes nonlinear inelastic deformation, internal heat generation (strain-heating), thermal expansion, thermoelastic coupling and temperature dependent material property, is considered. A detailed study is given for a special material model (nonlinear Maxwell), which approximately characterizes some polymers and metallic materials at elevated temperatures. For a quasi-static nonlinear problem, significant temperature increases are found for aluminum by means of an iterative numerical solution. Further investigations by a linearized perturbation technique shows that a thermal instability (runaway) may occur, depending on the choice of material, steady state values of stress and temperature and the length of the bar, but not on the magnitude or the form of the disturbance. The phenomenon of thermal runaway is the result of mutual feedback between strain-heating and the temperature dependent (inelastic) material properties. If either effect is not included in an analysis, the possibility of thermal runaway will not be explored and an unexpected failure may occur. The effect on the thermal runaway due to inertia, thermal expansion and thermoelastic coupling has been shown to be relatively minor for the considered one-dimensional problem. However, these effects could be important for more complicated loading, geometric situations and flexible structures (e.g. trusses, beams, plates). Such cases require further study.

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Table 1 Material Properties for Aluminum and Other Parameters

density ρ (Kg/m^3)	2707	creep activation constant B (°K)	13603
heat capacity c (J/Kg°K)	896	length L (m)	1
thermal conductivity k (W/m°K)	222	steady state stress σ_o (Pa)	3.5×10^7
Youngs' Modulus E (Pa)	5.52×10^{10}	reference temperature T_o (°K)	533
stress power parameter n	4.55	characteristic time t_o (second)	10
pre-exponential constant A_o	7.836×10^{-27}	mechanical disturbance σ_1/σ_1	.05
thermal expan. coeff. α (1/°K)	2.3×10^{-5}	thermal disturbance T_1/T_o	.01

Figure Captions

Fig. 1 Nonlinear problem - temperature increase with t at $x = 0$, L/2, 3L/4 for aluminum

Fig. 2 Nonlinear problem - temperature variation with x for $t = 250$, 500, 750, 1000 sec. for aluminum

Fig. 3 Linearized problem - stability diagram of steady state stress vs. steady state temperature for an aluminum bar 1 meter long

Fig. 4 Linearized problem - temperature increment due to mechanical disturbance

Fig. 5 Linearized problem - temperature increment due to thermal disturbance

Figure Captions

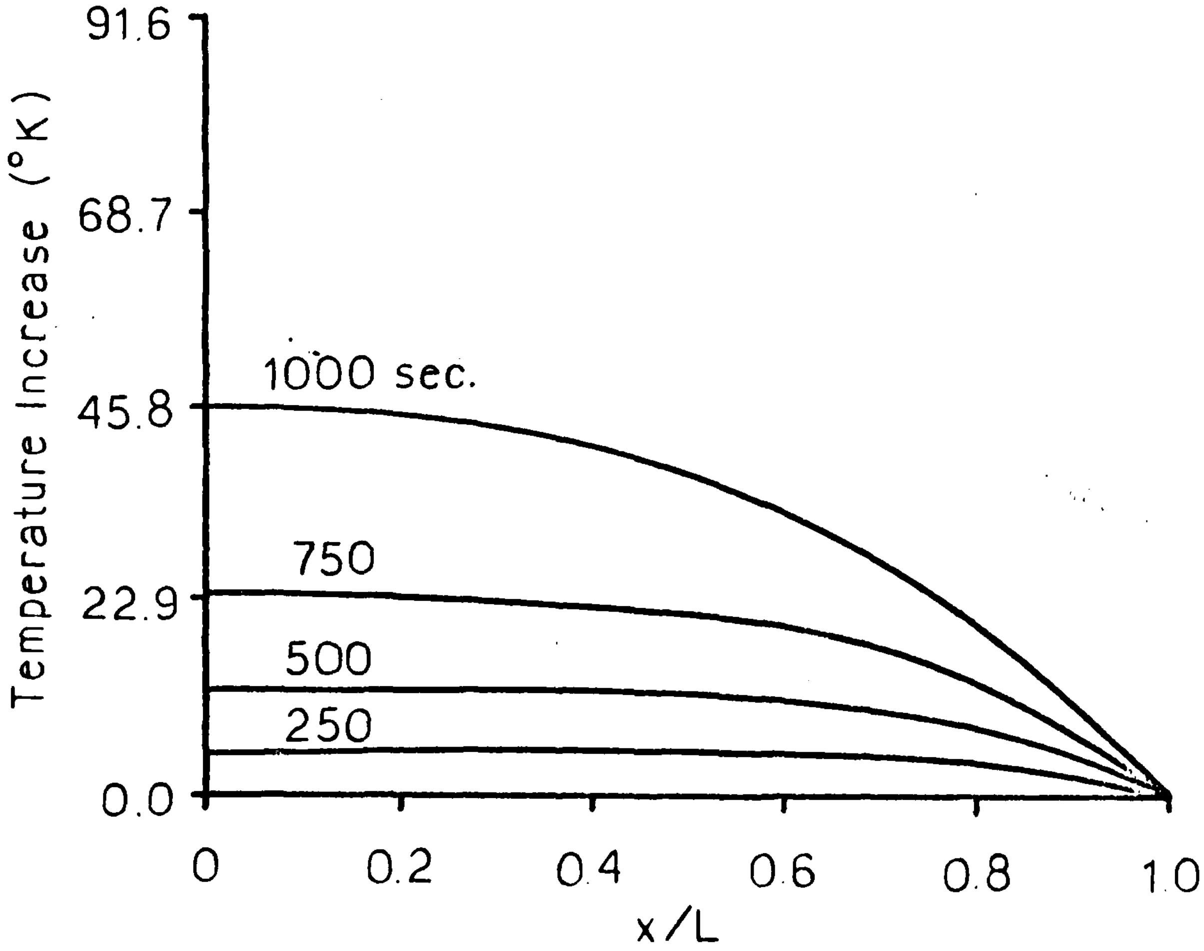
Fig. 1 Nonlinear problem - temperature increase with t at $x = 0, L/2, 3L/4$ for aluminum

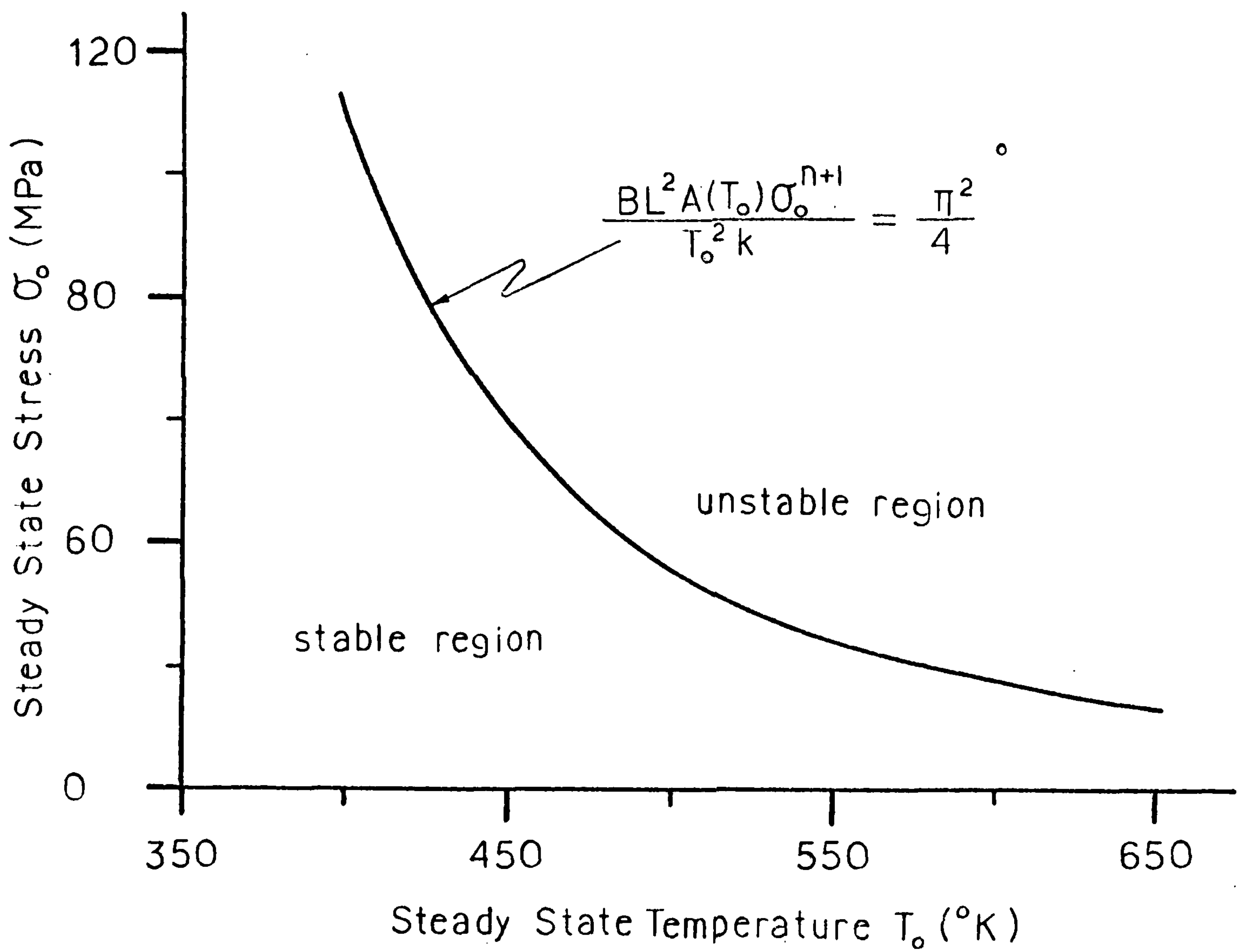
Fig. 2 Nonlinear problem - temperature variation with x for t = 250, 500, 750, 1000 sec. for aluminum

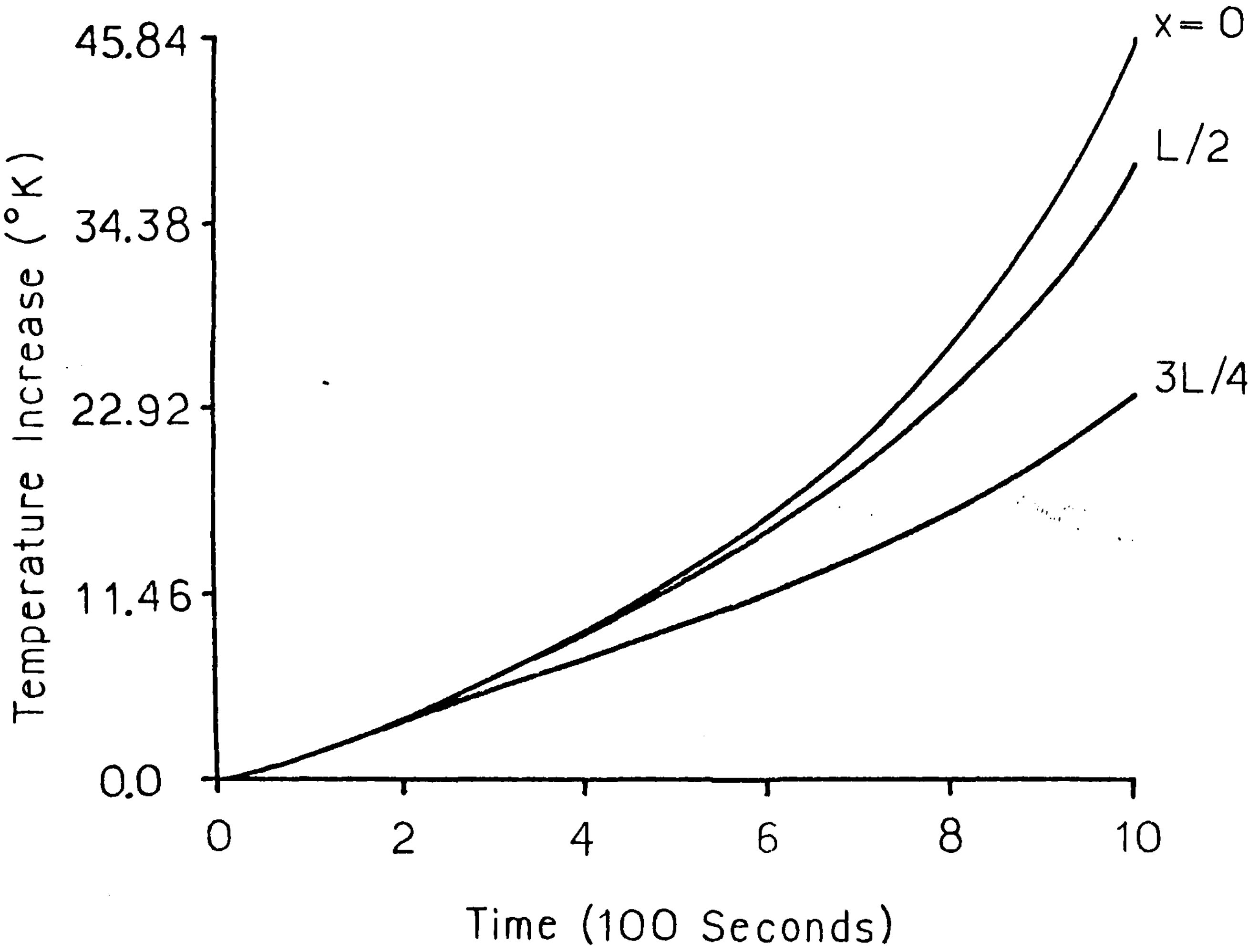
Fig. 3 Linearized problem - stability diagram of steady state stress vs. steady state temperature for an aluminum bar 1 meter long

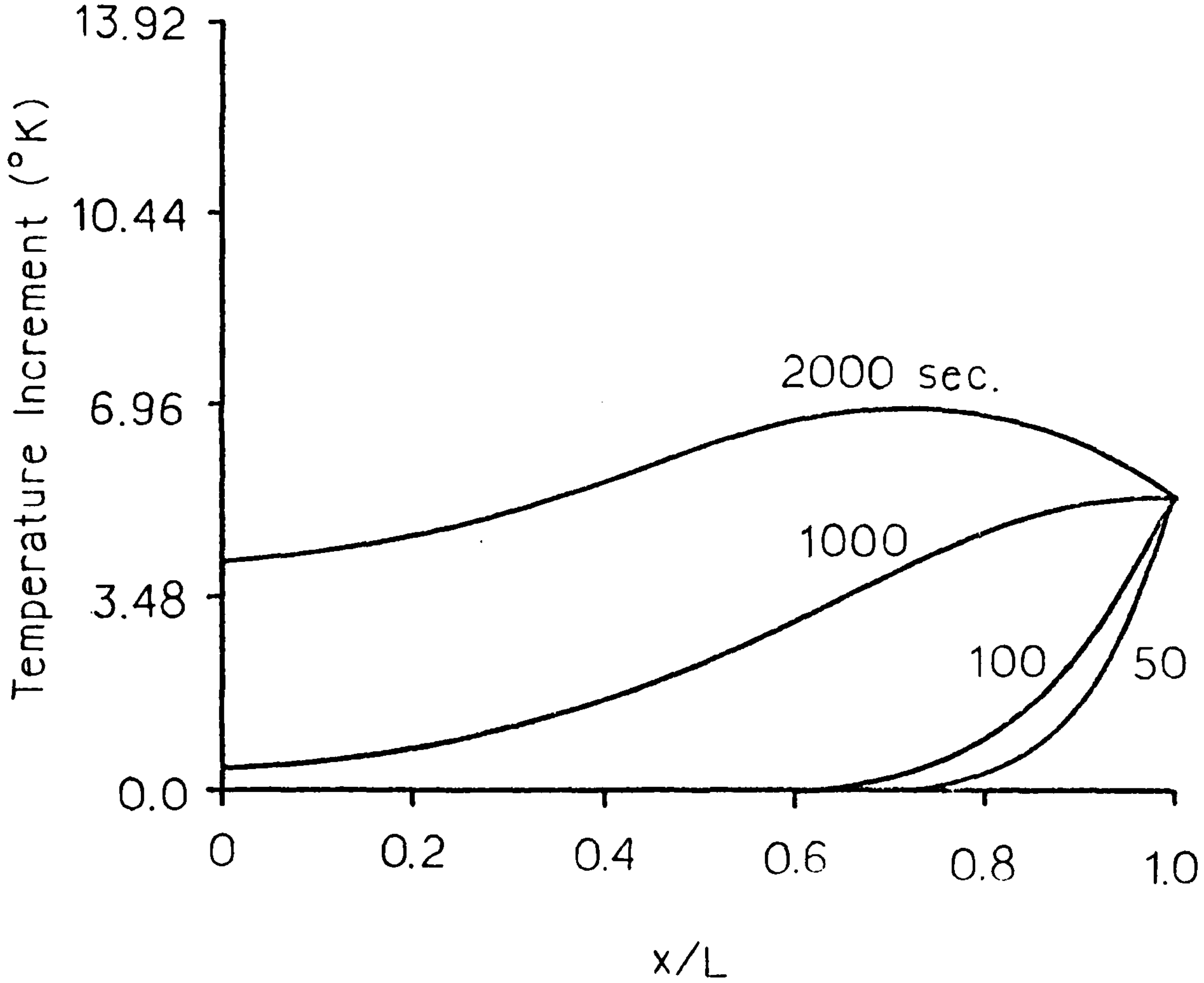
Fig. 4 Linearized problem - temperature increment due to mechanical disturbance

Fig. 5 Linearized problem - temperature increment due to thermal disturbance

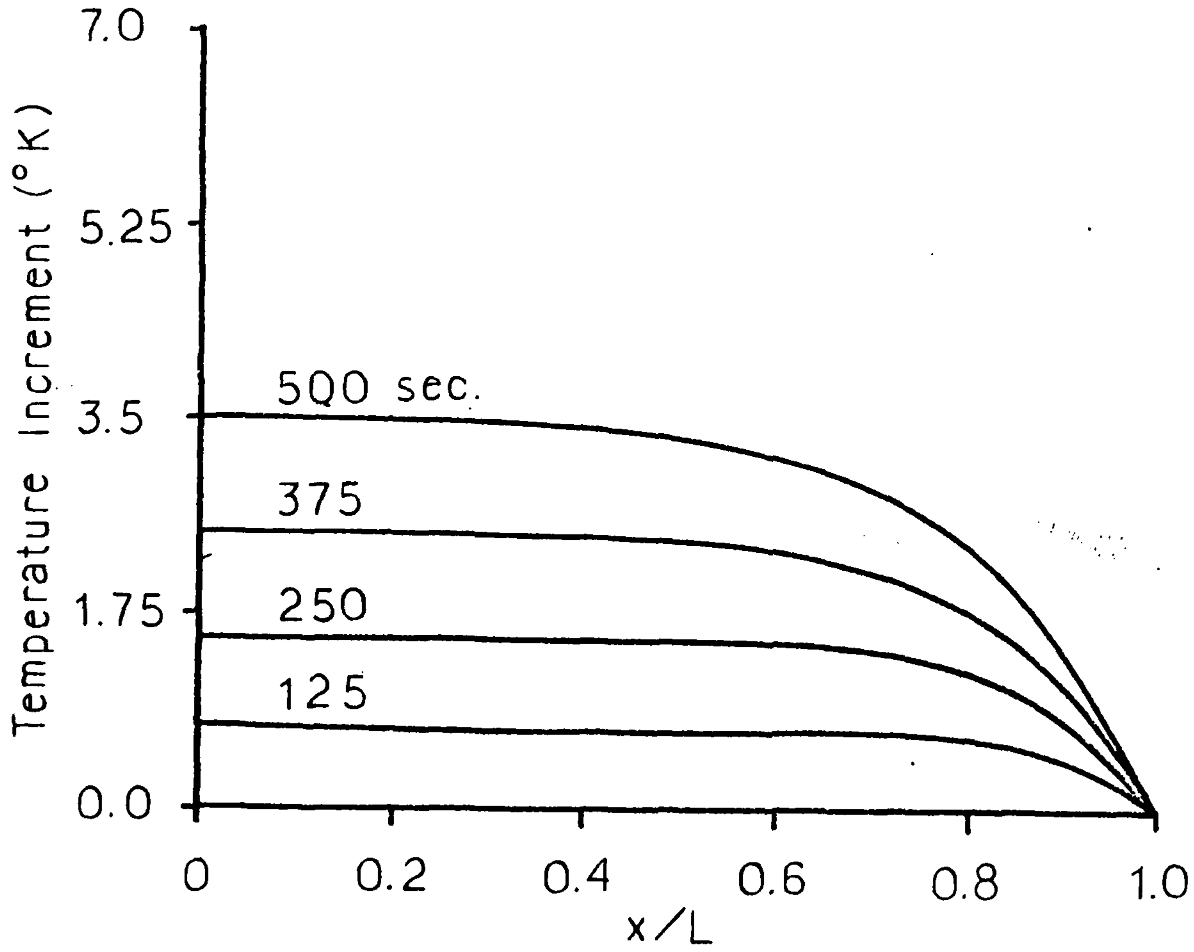

91.6
68.7
45.8
22.9
0.0
Temperature Increase (°K)
1000 sec.
750
500
250
0
0.2
0.4
0.6
0.8
1.0
x/L


120
80
60
0
350
450
550
650
Steady State Stress σ_o (MPa)
Steady State Temperature T_o (°K)
$\frac{BL^2 A(T_o)\sigma_o^{n+1}}{T_o^2 k} = \frac{\pi^2}{4}$
unstable region
stable region


x= 0
L/2
3L/4
45.84
34.38
22.92
11.46
0.0
Temperature Increase (°K)
0
2
4
6
8
10
Time (100 Seconds)



Temperature Increment (°K)
13.92
10.44
6.96
3.48
0.0
2000 sec.
1000
100
50
0
0.2
0.4
0.6
0.8
1.0
x/L



7.0
5.25
3.5
1.75
0.0
Temperature Increment (°K)
500 sec.
375
250
125
0
0.2
0.4
0.6
0.8
1.0
x/L

END
DATE
FILM
4-88
DTIC